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# Discrete Vortex Modelling of Turbulent Flow in Two Dimensions

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**Abstract:** There has been growing need to characterize the turbulent flow through a simplified model. This paper reports the characterization of turbulent flow in two dimensions using discrete vortex modelling. The free surface flow in channel is considered as a case study. Appropriate flow charts and FORTRAN source codes were developed to solve the main governing equations of fluid flow. The Reynolds number which is the control parameter from 100,000 (experimental transition point) at an interval of 10,000 to 1,000,000 is used and the result is displayed using Microsoft Excel Graph. After the four stages of transition process due to Helmholtz instability, the turbulent region is reached, which is characterized by irregularity, low momentum diffusion, high momentum convection, and rapid variation of velocity. The result has shown that turbulent flow can be characterized with ease by using discrete vortex modelling.

**Key words**: Turbulent flow, discrete vortex modeling, free surface flow in channel, Helmholtz instability, irregularity, high momentum convection.

## INTRODUCTION

The two different types of real fluid flow are laminar flow and turbulent flow. The laminar flow is a flow over a smooth surface with vorticity that appear highly ordered. Turbulent flow is a flow regime characterized by chaotic, disordered and stochastic property changes. Fluid flow that is slow and has inertial effect tends to be laminar. As it speeds up, a transition takes place and its crinkles up into complicated random turbulent flow. Turbulence is the time dependent chaotic behaviour seen in many fluid flows. Turbulence is a different type of fluid flow to laminar flow, hence it is desirable to be able to quantity under what conditions it occurs. The (dimensionless) Reynolds number gives quantitative indication of laminar to turbulent transition and characterizes whether the flow conditions lead to laminar or turbulent flow. Transition to turbulence can occur over a range of Reynolds numbers, depending on many factors such as level surface roughness, heat transfer, vibration, noise

and other disturbances. For flow in a pipe, the characteristic length is the pipe diameter while for flow over a flat plate, the characteristic length is usually the length of the plate and the characteristic velocity is the free stream velocity.

Diffusion methods such as core-spreading techniques, hair pin-removal, particle strength, exchange, vorticity redistribution method and random walk method have been developed to characterize fluid flow. The random walk method was introduced by Chorin (1978) to study slightly viscous flow. Morchore and Pulvient (1982), Goodman (1987) and Long (1988) have shown that for flow in free-space, the random walk solution converges to that of the Navier-Stokes equations as the number of vorticies is increased. Gagnon and Mercader (1996) used the random walk method to compute the starting flow behind a two-dimensional step. A two-dimensional random vortex method is used by Gagnon (1993) to simulate the flow over a single back-facing step and a double symmetrical backward-facing step. Cheer (1989) has implemented the random walk method for flows over a cylinder. Lewis (1990) has used the random walk method for flow over airfoil cascades. Abdolhosseini (1996) studies the turbulent statistics in a uniformly sheared flow with a two-dimensional using random walk method. Chui (1993) used the random walk method to study thermal boundary layers. Adegbola and Salau (2011) has implemented the random walk method to characterize the fluid flow into laminar, transition and turbulent region.

The random walk method has several advantages. It requires simple algorithms and it can easily handle flows around complicated boundaries. The method also conserves the total circulation. The free surface flow in channel is used as a case study in this paper. The objective of the work is to develop a simplified model for turbulent flow through the use of discrete vortex method. The study intends to discover the unique features of the turbulent flow. Turbulence has been described as "the most important unsolved problem of classical physics, hence there has been growing need to develop a simplified model which can be used to predict the fluid flow that moves in an erratic or random motion (turbulent flow). This paper reports a "random walk model" for predicting turbulent flow in two dimensions through the use of "discrete vortex modelling".

## MODEL FORMULATION

The boundary layer flow can be approximated by placing at appropriate locations some vortices in a parallel flow. This forms the basis of the vortex element method. The principle involved is to subject the entire free vortex elements to small random displacements, which produce a scatter equivalent to the diffusion of vorticity in the continuum, which we are seeking to represent.

Turbulent flow is always three-dimensional. However, when the equations are time aver-aged, it can be treated as two-dimensional.

**Diffusion of a point vortex in two-dimensional flow**

The motion of a diffusing vortex of initial strength (**Γ**) centered on the origin of the (r,) plane is described by the diffusion equations.

$$\frac{d\omega}{dt} = v\,\bar{\nabla}^2\,\omega \qquad (1)$$

from which we may obtain the wall solution in space and time.

$$\omega(r,t) = \frac{\Gamma}{4\pi vt}\, e^{\left(-r^2/4vt\right)} \qquad (2)$$

Vorticity strength is a function of radius r and time t, where π is the ratio of the circumference of a circle to its diameter and v is the kinematic viscosity.

For a vortex of unit strength split in N elements, let us assume that n vortex elements are scattered into the small area rΔ$\theta$Δr after time t. The total amount of vorticity in the element of area is:

$$P_v = \frac{n}{N} = \left[\frac{1}{4\pi vt}\, e^{\left(-r^2/4vt\right)}\right] r\Delta\theta\Delta r \qquad (3)$$

It is obvious from symmetry that scattering in the $\theta$ direction ought to be done with equal probability. Hence $\theta_i$ values may be defined independently of $r_i$ values by the equations:

$$\theta_i = 2\pi Q_i \qquad (4)$$

Where $\theta_i$ is a random number within the range $0 < \theta_i < 1$. The probability P that on element will lie within a circle of radius r is given by the equation:

$$P = 1 - e^{\left(-r^2/4vt\right)} \qquad (5)$$

Hence for the vortex element (5) becomes

$$P_i = 1 - e^{\left(-r_1^2/4vt\right)} \qquad (6)$$

From which we may obtain it radial shift

$$r_i = \left\{4vt\, In\left(\frac{1}{1-P_i}\right)\right\}^{1/2} \qquad (7)$$

**Random Number Generation**

Algorithms were developed to produce long sequences of apparently random results which are fact completely determined by a shorted initial value known as a seed.

**Diffusion over a series of Time steps**

The displacements of element i during time Δt for diffusion over a succession of small time increments is given by:

$$\Delta\theta_i = 2\pi Q_i \qquad (8)$$

$$\Delta r_i = \left\{4v\Delta t ln\left(\frac{1}{1-P_i}\right)\right\}^{1/2} \qquad (9)$$

After the increment Δt, the new co-ordinate location ($x_i^{'}, y_i^{'}$) of the ith element will become:

$$x_1^{'} = x_1 + \Delta r_i \cos \Delta \theta_i \quad (10)$$

$$y_1^{'} = y_1 + \Delta r_i \sin \Delta \theta_i \quad (11)$$

where $x_i$ and $y_i$ are the old x –coordinate the ith element and old y - coordinate of the ith element respectively. The displacement of the ith element from the origin is given by the equation.

$$D_i = \sqrt{\left(x_1^{'} - x_0\right)^2 + \left(y_1^{'} - y_0\right)^2} \quad (12)$$

Where $x_o$ and $y_o$ are the origin.

**Boundary layers by discrete vortex modelling**

Convective motions were completely ignored for the diffusing point which we have just considered, an assumption which is permissible in view of symmetry in these special cases and justified for very low Reynolds number. Boundary layer flows on the other hand are more complex involving two additional features:

i. Externally imposed convention due to the main stream U, the significance of which is determined by the body scale Reynolds number (UL/v).
ii. Continuous creation of vorticity at the contact surface between fluid and wall, replacing the vorticity removed by diffusion and convection.

**Vorticity Creation**

The treatment of viscous boundaries in a vortex method is done by defining at the boundary, special sheets of vorticity which stay at the boundary and diffuse their vorticity to free elements.

**Random walk method**

Application of the random walk will result in the loss of half of the newly created vorticity due to diffusion across the wall and therefore out of the active flow domains. Since vorticity creation, diffusion and convection are being considered independently and in sequence in the computational scheme, vorticity must be considered. A single strength sheet is used by making sure that vortices which attempt to cross the wall are bounced back by assigning the value $y_i$ = abs ($y_i$).

**Selection of element size and time step**

A reasonable approach to the selection of an appropriate time step Δt is to focus attention on the average displacements of the discrete vortices due to convection and diffusion.

The average convective displacement may be approximated by:

$$\delta_c = \frac{1}{2} U \Delta t \quad (13)$$

The average diffusive displacement is

$$\delta_D = \sqrt{(4v\Delta t\ In2)} \quad (14)$$

In order to maintain equal discretisation of the fluid motion due to convection and diffusion, we equate δc and $\delta_D$ resulting in the expression:

$$\Delta t = \frac{16LIn2}{URe} \quad (15)$$

Where Re = UL/v is the plate Reynolds number.

It is more reasonable to select the surface element size at twice δc leading to:

$$\Delta s = U\Delta t = \frac{16LIn2}{Re} \quad (16)$$

The required number of surface elements for satisfactory discretisation of the plate is given by:

$$M = \frac{L}{\Delta s} = \frac{Re}{16\ In\ 2} \quad (17)$$

It is obvious that enforcing equal discretisation scales $\delta_o$ and $\delta_D$ for convection and diffusion will lead to computational difficulties at large Reynolds number. The large Reynolds number will impose severe pressure upon computational requirements. Hence practical computational limitation will rule out vortex modelling for typical engineering system Reynolds numbers if we attempt to impose the constraint $\delta_D = \delta_c$

**Some considerations for high Reynolds numbers**

The difficulties for high Reynolds numbers can be eased by selecting different time steps for diffusion ($\Delta t_D$) and convection ($\Delta t_c$). Since convection now dominates the flow, it is better to select the scale of convection displacements through:

$$K = \frac{\delta_c}{\Delta_s} \quad (18)$$

where previously K has been set to 0.5. The convective time step then follows from (13)

$$\Delta t_c = \frac{2K\Delta_s}{\mu} = \frac{2K}{M}\left(\frac{\iota}{\mu}\right) \quad (19)$$

The average random walk diffusive displacement over the same interval follows from (14), namely

$$\frac{\delta_D}{\Delta s} = \sqrt{\left(\frac{8\ MK\ In2}{Re}\right)} \quad (20)$$

Although, it would be perfectly to perform both the convection and random walk processes over the same time step $\Delta t_c$, a saving in computational effort could be achieved by undertaking only one random walk for every $N_t$ convection steps with:

$$\Delta t_D = N_t \Delta t_c \quad (21)$$

**Calculation of Velocity Profile**

The relationship between average displacements of the ith element from the origin to the number of time steps is given by:

$D_{av} \propto t_H$

$$\therefore\ D_{av} = Kt^H \qquad (22)$$

K and H are diffusion constant and Index respectively. From (23), taking the log of both sides

$$\log D_{av} = Hlogt + Log\ K \qquad (23)$$

Equation on a straight line is given as:

$$y = mx + c \qquad (24)$$

Comparing (23) and (24)

y = log $D_{av}$, c = log K, x = log t and m = N

Having developed the governing equations for the fluid flow using the Reynolds number as the tuning parameter, the algorithms is formulated for the model the transition region to turbulent region. which is illustrated by the flow chart. The Reynolds number of 100,000 (Experimental transition points) at an interval of 10,000 to 1,000,000 is used. The flow chart is used in writing the FORTRAN program, the program is then run to generate the desired output. The result obtained was used to plot the graphs through the use of Microsoft Excel.

## RESULTS AND DISCUSSION

Table 1 shows the Reynolds number, Time increment, number of time steps, number of elements or trials, log of average distance against log of time steps and index (velocity). The index is the slope obtained from the graph of log of average distance against log of time steps. There is fluctuation of index (velocity) from

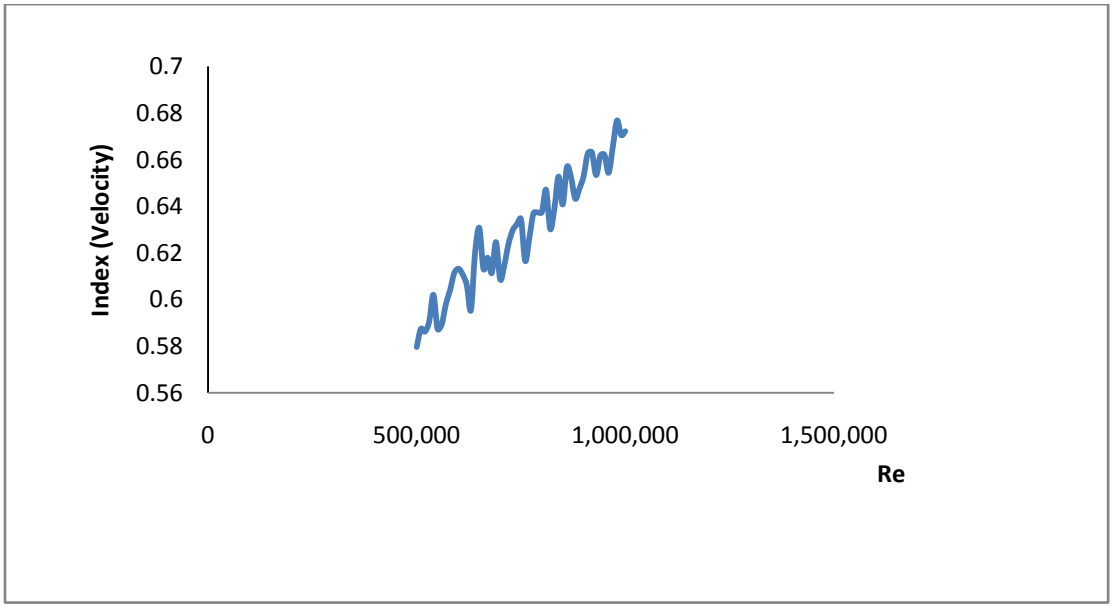


**Figure 3: Depiction of Index (Velocity) against Reynolds Number for Turbulent Region**

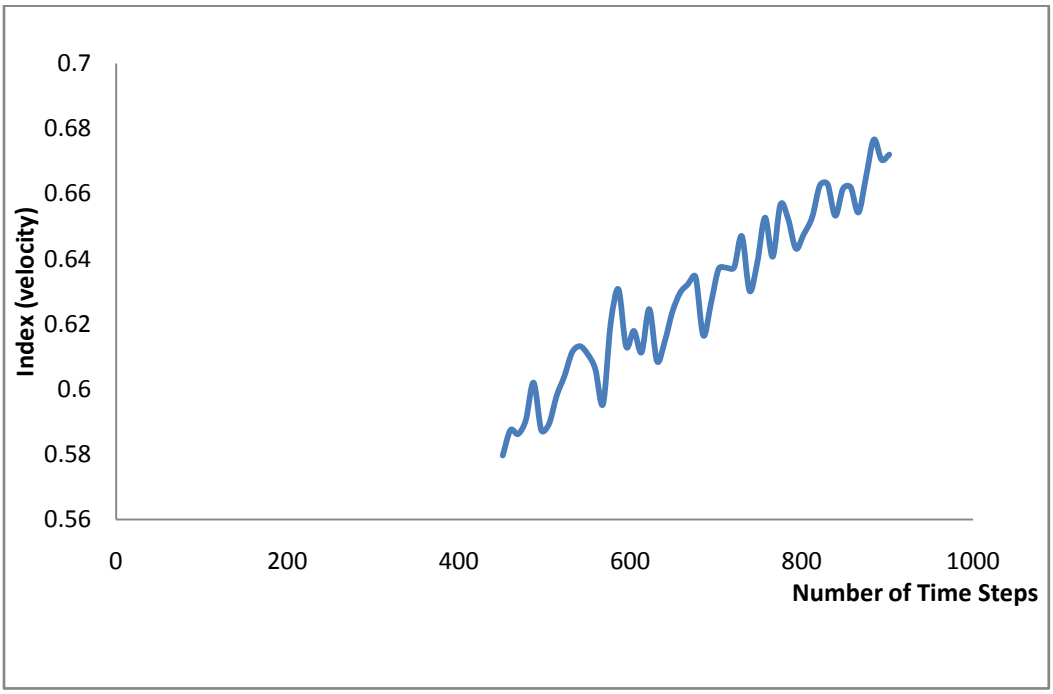


**Figure 4: Depiction of Index (Velocity) against number of time steps for Turbulent Region**

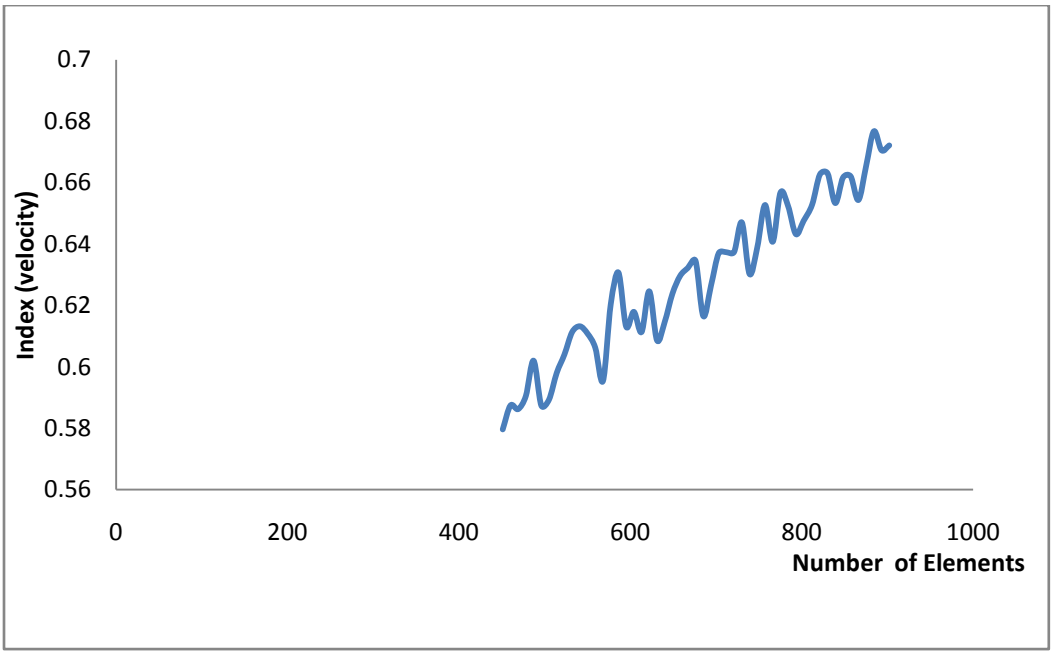


**Figure 5: Depiction of Index (velocity) against Number of elements for Turbulent Region**

The initial stage of the natural transition process (receptive phase) consist of the transformation of external disturbances in the outer free stream flow over the boundary layer into internal instability oscillations within the boundary layer from the Reynolds number of 100,000. The second stage of the process is the exponential growth of the few unstable disturbances from Reynolds number of 180,000. In the third stage from Reynolds number of 290,000, the amplitude of the disturbances now large enough to introduce non-linearlity effects. Due to the distortion of the boundary layer, inflexional mean profiles develop and a fourth stage is reached from Reynolds number of 420,000 where the boundary layer becomes unstable to high frequency disturbances. Finally, an explosive growth of these high frequency disturbances initiate the fifth and final phase from Reynolds number of 500,000 (the breakdown into turbulence). Hence it can be perceived how transition of a boundary layer from laminar to turbulent motion take place at very high Reynolds number, bearing in mind the Kelvin-Helmholtz instability. The graphs (fig 3, fig 4 and fig 5) show distinct region of turbulent flow which have the following features.

- ***Irregularity:*** Turbulent flow is highly irregular and chaotic.
- ***Diffusion and convection:*** Turbulent flow has low momentum diffusion and high momentum convection
- ***Unsteady and Non uniform:*** There is rapid variation of velocity in space and time.
- ***Energy Cascade:*** It is made up of super position of a spectrum of velocity fluctuation and eddies an over mean flow. Their hierarchy can be described by energy spectrum that measures the energy in velocity fluctuations of wave number.

## CONCLUSION

The study has explored discrete vortex model to characterize the turbulent flow in two dimensions. The transition to turbulent motion takes place at high Reynolds number at the fifth stage due to Helmholtz instability. The turbulent region is characterized by irregularity, low momentum diffusion, high momentum convection and rapid variation of velocity.

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**Table 1: Parameters Estimation for Transition and Turbulent Regions**

| S/N | Reynolds number (Re) | Time Increment ($\Delta t$) | Number of time steps (N) | Number of elements or trials (M) | Log of average distance against log of time steps (y = mx + c) | Index (m) |
|---|---|---|---|---|---|---|
| 1 | 100,000 | 0.01109 | 90 | 90 | y = 0.4681x + 0.01918 | 0.4681 |
| 2 | 110,000 | 0.01008 | 99 | 99 | y = 0.4542 x + 0.2166 | 0.4542 |
| 3 | 120,000 | 0.00924 | 108 | 108 | y = 0.4906 x +0.1595 | 0.4906 |
| 4 | 130,000 | 0.00853 | 117 | 117 | y =0.4423 x + 0.0.2401 | 0.4423 |

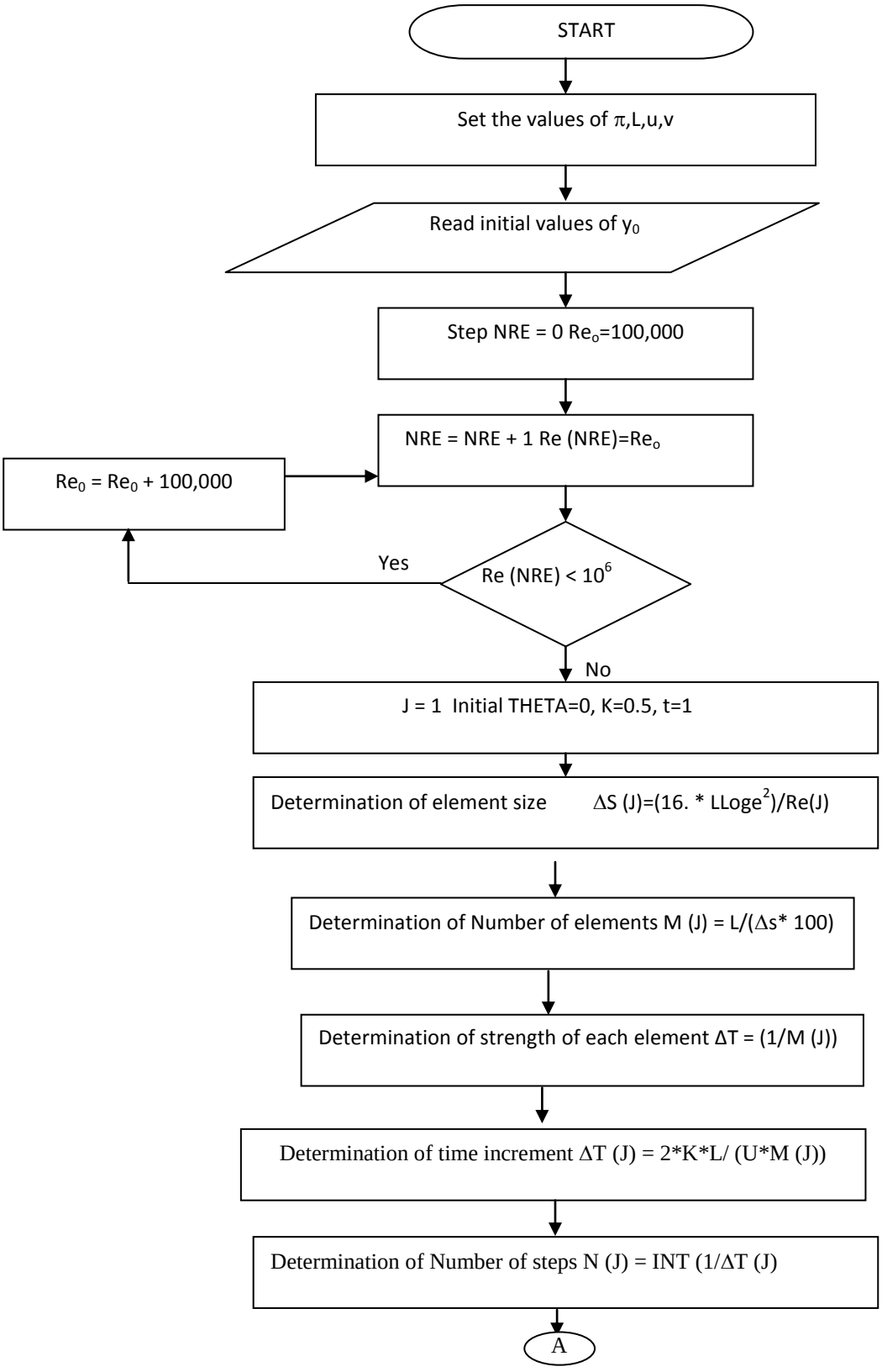
| | | | | | | |
|---|---|---|---|---|---|---|
| 5. | 140,000 | 0.00792 | 126 | 126 | y = 0.4773 x +0.177 | 0.4773 |
| 6 | 150,000 | 0.00739 | 135 | 135 | y = 0.4655 x + 0.182 | 0.4655 |
| 7 | 160,000 | 0.00693 | 144 | 144 | y = 0.4821 x +0.1588 | 0.4821 |
| 8 | 170,000 | 0.00652 | 153 | 153 | y =0.4755 x + 0.1779 | 0.4755 |
| 9 | 180,000 | 0.00616 | 162 | 162 | y = 0.5054 x + 0.1389 | 0.5054 |
| 10 | 190,000 | 0.00584 | 171 | 171 | y = 0.4808 x + 0.1572 | 0.4808 |
| 11 | 200,000 | 0.00555 | 180 | 180 | y = 0.4925 x + 0.1122 | 0.4925 |
| 12 | 210,000 | 0.00528 | 189 | 189 | y = 0.4936 x + 0.1423 | 0.4936 |
| 13 | 220,000 | 0.00504 | 198 | 198 | y = 0.4993 x + 0.155 | 0.4993 |
| 14 | 230,000 | 0.00482 | 207 | 207 | y = 0.5036 x + 0.1315 | 05036 |
| 15 | 240,000 | 0.00462 | 216 | 216 | y = 0.5332 x + 1896 | 0.5332 |
| 16 | 250,000 | 0.00444 | 225 | 225 | y = 0.5146 x + 0.125 | 0.5146 |
| 17 | 260,000 | 0.00427 | 234 | 234 | y = 0.5306 x + 0.097 | 0.5306 |
| 18 | 270,000 | 0.00411 | 243 | 243 | y = 0.5295 x + 0.1165 | 0.5295 |
| 19 | 280,000 | 0.00396 | 252 | 252 | y = 0.551 x + 0.655 | 0.551 |
| 20 | 290,000 | 0.00382 | 261 | 261 | y = 0.5348 x + 0.1018 | 0.5348 |
| 21 | 300,000 | 0.00370 | 271 | 271 | y = 0.5275 x + 0.0882 | 0.5275 |
| 22 | 310,000 | 0.00358 | 280 | 280 | y = 0.565 x + 0.051 | 0.565 |
| 23 | 320,000 | 0.00347 | 289 | 289 | y = 0.5459 x + 0.0544 | 0.5459 |
| 24 | 330,000 | 0.00336 | 298 | 298 | y = 0.5262 x + 0.189 | 0.5262 |
| 25 | 340,000 | 0.00326 | 307 | 307 | y = 0.5642 x + 0.0405 | 0.5642 |
| 26 | 350,000 | 0.00317 | 316 | 316 | y = 0.5384 x + 0.0661 | 0.5484 |

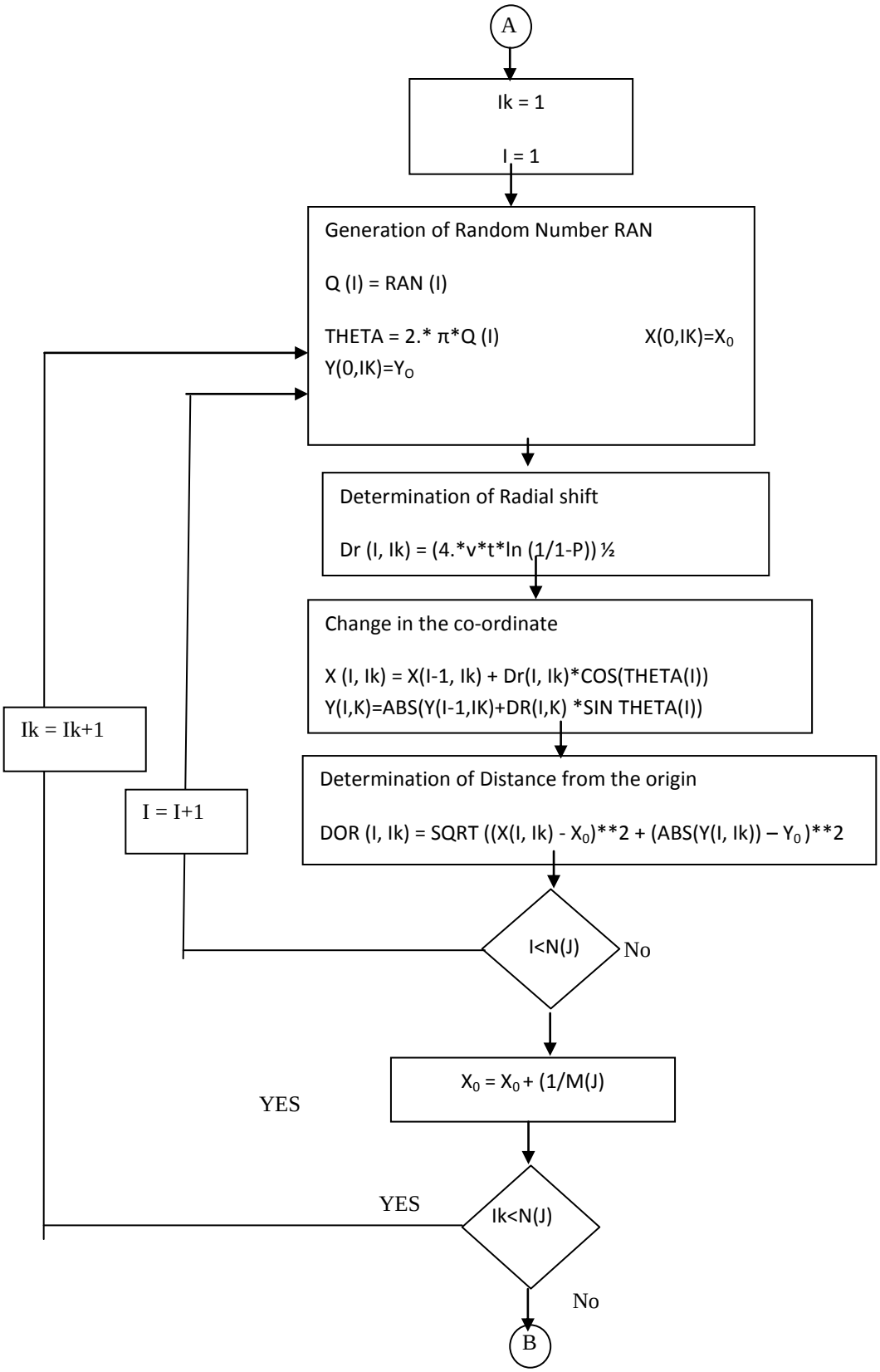
| | | | | | | |
|---|---|---|---|---|---|---|
| 27 | 360,000 | 0.00308 | 325 | 325 | y = 0.548 x + 0. 0661 | 0.548 |
| 28 | 370,000 | 0.00300 | 334 | 334 | y = 0.5633 x +0.0507 | 0.5633 |
| 29 | 380,000 | 0.00292 | 343 | 343 | y = 0.562 x + 0.0609 | 0.562 |
| 30 | 390,000 | 0.00284 | 352 | 352 | y = 0.584 x + 0.0132 | 0.584 |
| 31 | 400,000 | 0.00277 | 361 | 361 | y = 0.5674 x + 0.0685 | 0.5674 |
| 32 | 410,000 | 0.00270 | 370 | 370 | y = 0.5549 x + 0.0514 | 0.5549 |
| 33 | 420,000 | 0.00264 | 379 | 379 | y = 0.5632 x + 0.0438 | 0.5632 |
| 34 | 430,000 | 0.00258 | 388 | 388 | y = 0.5628 x + 0.0454 | 0.5628 |
| 35 | 440,000 | 0.00252 | 397 | 397 | y =0.5522 x + 0.0641 | 0.5522 |
| 36 | 450,000 | 0.00246 | 406 | 406 | y = 0.5847 x + 0.0133 | 0.58 |
| 37 | 460,000 | 0.00241 | 415 | 415 | y = 5696 x + 0.0332 | 0.5696 |
| 38 | 470,000 | 0.00236 | 424 | 424 | y = 0.5663 x + 0.0354 | 0.5663 |
| 39 | 480,000 | 0.00231 | 433 | 433 | y = 0.5797 x – 0.0231 | 0.5797 |
| 40 | 490,000 | 0.00226 | 441 | 441 | y = 0.5927 x – 0.017 | 0.5927 |
| 41 | 500,000 | 0.00222 | 451 | 451 | y = 0.5796 x + 0.0211 | 0.5796 |
| 42 | 510,000 | 0.00217 | 460 | 460 | y = 0.5874 x + 0.0086 | 0.5874 |
| 43 | 520,000 | 0.00213 | 469 | 469 | y = 0.5862 x – 0.0027 | 0.5862 |
| 44 | 530,000 | 0.00209 | 478 | 478 | y = 0.5905 x + 0.0202 | 0.5905 |
| 45 | 540,000 | 0.00205 | 487 | 487 | y = 0.602 x – 0.0287 | 0.602 |
| 46 | 550,000 | 0.00202 | 496 | 496 | y = 0.5874 x + 0.0153 | 0.5874 |
| 47 | 560,000 | 0.00198 | 505 | 505 | y = 0.5893 x + 0.0085 | 0.5893 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 48 | 570,000 | 0.00195 | 514 | 514 | y = 0.598 x – 0.0155 | 0.598 |
| 49 | 580,000 | 0.00191 | 523 | 523 | y = 0.604 x – 0.0226 | 0.604 |
| 50 | 590,000 | 0.00188 | 532 | 532 | y = 0.6113 x – 0.0326 | 0.6113 |
| 51 | 600,000 | 0.00185 | 541 | 541 | y = 0.6132 x – 0.0434 | 0.6132 |
| 52 | 610,000 | 0.00182 | 550 | 550 | y = 0.6108 x – 0.0307 | 0.6108 |
| 53 | 620,000 | 0.00179 | 559 | 559 | y = 0.6062 x – 0.0253 | 0.6062 |
| 54 | 630,000 | 0.00176 | 568 | 568 | y = 0.5954 x – 0.007 | 0.5954 |
| 55 | 640,000 | 0.00173 | 577 | 577 | 0.6203 x – 0.073 | 0.6203 |
| 56 | 650,000 | 0.00171 | 586 | 586 | y = 0.6307 x – 0.0713 | 0.6307 |
| 57 | 660,000 | 0.00168 | 595 | 595 | y = 0.6131 x – 0.0494 | 0.6131 |
| 58 | 670,000 | 0.00166 | 604 | 604 | y = 0.6179 x – 0.042 | 0.6179 |
| 59 | 680,000 | 0.00163 | 613 | 613 | y = 0.6113 x – 0.0362 | 0.6113 |
| 60 | 690,000 | 0.00161 | 622 | 622 | y = 0.6246 x – 0.0687 | 0.6246 |
| 61 | 700,000 | 0.00158 | 631 | 631 | y = 0.6086 x – 0.0407 | 0.6086 |
| 62 | 710,000 | 0.00156 | 640 | 640 | y = 0.6146x – 0.0582 | 0.6146 |
| 63 | 720,000 | 0.00154 | 649 | 649 | y = 0.6238x – 0.0637 | 0.6238 |
| 64 | 730,000 | 0.00152 | 658 | 658 | y = 0.6296 x – 0.0665 | 0.6296 |
| 65 | 740,000 | 0.00150 | 667 | 667 | y = 0.6322 x – 0.0563 | 0.6322 |
| 66 | 750,000 | 0.00148 | 676 | 676 | y = 0.6345 x – 0.0827 | 0.6345 |
| 67 | 760,000 | 0.00146 | 685 | 685 | y = 0.6165 x – 0.0471 | 0.6165 |
| 68 | 770,000 | 0.00144 | 694 | 694 | y = 0.6264 x – 0.0756 | 0.6264 |
| 69 | 780,000 | 0.00142 | 703 | 703 | y = 0.637 x – 0.0898 | 0.637 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 70 | 790,000 | 0.00140 | 712 | 712 | y = 0.6373 x – 0.0768 | 0.6373 |
| 71 | 800,000 | 0.00139 | 721 | 721 | y = 0.6374 x – 0.0848 | 0.6374 |
| 72 | 810,000 | 0.00137 | 730 | 730 | y = 0.647 x – 0.1019 | 0.647 |
| 73 | 820,000 | 0.00135 | 739 | 739 | y = 0.6302 x – 0.0814 | 0.6302 |
| 74 | 830,000 | 0.00134 | 748 | 748 | y = 0.6389 x – 0.1007 | 0.6389 |
| 75 | 840,000 | 0.00132 | 757 | 757 | y = 0.6527 x – 0.1166 | 0.6527 |
| 76 | 850,000 | 0.00130 | 766 | 766 | y = 0.6407 x – 0.1038 | 0.6407 |
| 77 | 860,000 | 0.00129 | 775 | 775 | y = 0.6567 x – 0.1206 | 0.6567 |
| 78 | 870,000 | 0.00127 | 784 | 784 | y = 0.6523 x – 0.1211 | 0.6523 |
| 79 | 880,000 | 0.00126 | 793 | 793 | y = 0.6432 x – 0.1078 | 0.6432 |
| 80 | 890,000 | 0.00125 | 802 | 802 | y = 0.6475 x – 0.1076 | 0.6475 |
| 81 | 900,000 | 0.00123 | 812 | 812 | y = 0.6528 x – 0.1325 | 0.6528 |
| 82 | 910,000 | 0.00122 | 821 | 821 | y = 0.6626 x – 0.466 | 0.6626 |
| 83 | 920,000 | 0.00121 | 830 | 830 | y = 0.663 x – 0.1417 | 0.663 |
| 84 | 930,000 | 0.00119 | 839 | 839 | y = 0.6533 x – 0.1233 | 0.6533 |
| 85 | 940,000 | 0.00118 | 848 | 848 | y = 0.6616 x – 0.1317 | 0.6616 |
| 86 | 950,000 | 0.00117 | 857 | 857 | y = 0.662 x – 0.384 | 0.662 |
| 87 | 960,000 | 0.00116 | 866 | 866 | y = 0.6543x – 0.114 | 0.6543 |
| 88 | 970,000 | 0.00114 | 875 | 875 | y = 0.6654 x – 0.1337 | 0.6654 |
| 89 | 980,000 | 0.00113 | 884 | 884 | y = 0.6767 x – 0.0417 | 0.6767 |
| 90 | 990,000 | 0.00112 | 893 | 893 | y = 0.6705 x – 0.1579 | 0.6705 |
| 91 | 1,000,000 | 0.00111 | 902 | 902 | y = 0.6721 x – 0.1612 | 0.6721 |

## APPENDIX A


START
Set the values of π,L,u,v
Read initial values of $y_0$
Step NRE = 0 $Re_o$=100,000
NRE = NRE + 1 Re (NRE)=$Re_o$
$Re_0 = Re_0 + 100,000$
Yes
Re (NRE) < $10^6$
No
J = 1 Initial THETA=0, K=0.5, t=1
Determination of element size ΔS (J)=(16. * LLoge$^2$)/Re(J)
Determination of Number of elements M (J) = L/(Δs* 100)
Determination of strength of each element ΔT = (1/M (J))
Determination of time increment ΔT (J) = 2*K*L/ (U*M (J))
Determination of Number of steps N (J) = INT (1/ΔT (J)
A


A
Ik = 1
I = 1
Generation of Random Number RAN
Q (I) = RAN (I)
THETA = 2.* π*Q (I)
X(0,IK)=X0
Y(0,IK)=YO
Determination of Radial shift
Dr (I, Ik) = (4.*v*t*ln (1/1-P)) ½
Change in the co-ordinate
X (I, Ik) = X(I-1, Ik) + Dr(I, Ik)*COS(THETA(I))
Y(I,K)=ABS(Y(I-1,IK)+DR(I,K) *SIN THETA(I))
Determination of Distance from the origin
DOR (I, Ik) = SQRT ((X(I, Ik) - X0)**2 + (ABS(Y(I, Ik)) – Y0 )**2
I<N(J)
No
YES
X0 = X0 + (1/M(J)
Ik<N(J)
YES
No
B
Ik = Ik+1
I = I+1

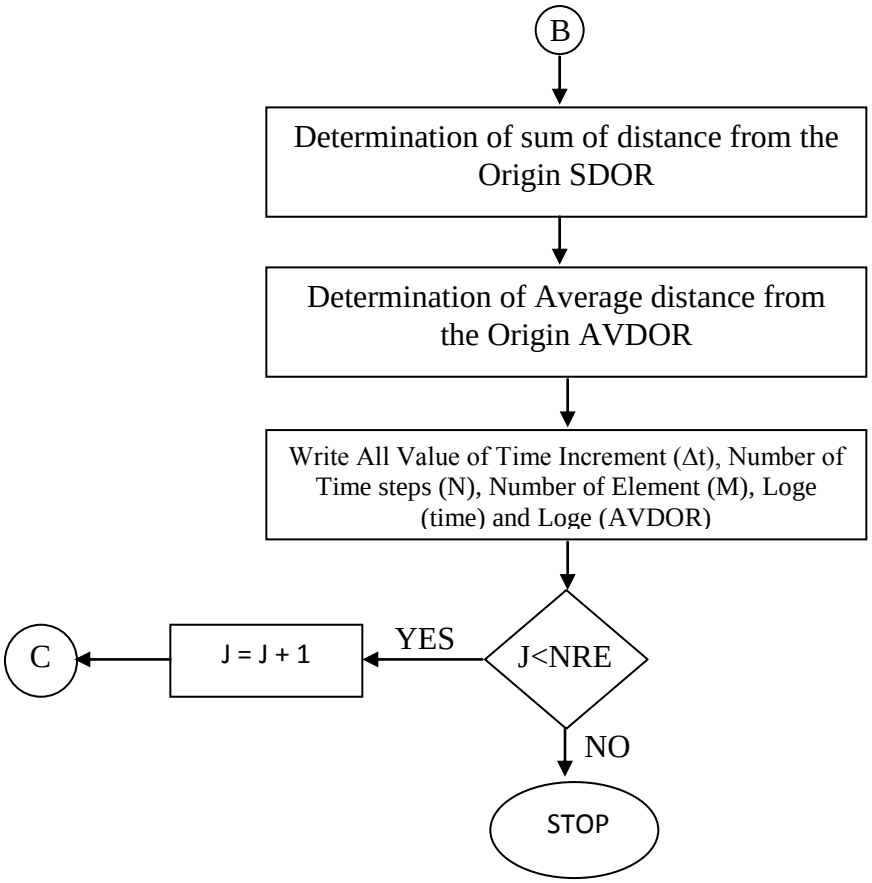



**Figure1: Discrete Vortex Model Flow Chart**