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# EVALUATION OF SENSITIVITY OF THREE SELECTED TWO-SAMPLE NONPARAMETRIC TESTS

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## Abstract
*In this paper, comparison of two sample tests, is motivated by the fact that in the test of significant difference between two independent samples, numerous methods can be adopted, each may lead to significant different results; this implies that wrong choice of test statistic could lead to erroneous conclusion. To prevent misleading information, there is a need for proper investigation of some selected methods for test of significant difference between variables/subjects most especially, independent samples. In this paper, Monte Carlo's Simulation techniques were used in the generation of data of two different distributions and varying sample sizes ranging from 5 to 100 which were repeated 30 times for each sample size. In the simulation, sample sizes 5, 10, 20, 30, 50 and 100 were considered. In the paper, data from a known family of distributions; Gamma (4, 0.3) and Weibull (7, 3) were used. This paper examines the sensitivity/efficiency of Mann-Whitney U test, Modified Mann-Whitney U test and Kolmogorov-Smirnov two-sample test to determine the most powerful test in terms of rejecting the null hypothesis when it is true. From the results, Mann-Whitney U-test was found to be the most powerful test in terms of rejecting the null hypothesis when it is true.*
**Keywords:** Independent Sample; Nonparametric test; two sample tests; Power of test

## Introduction
Non-parametric techniques do not rely on data belonging to any particular distribution. These include, among others, distribution free methods, which do not rely on assumptions that the data are drawn from a given probability distribution. As such, it is the opposite of parametric statistics. It includes non-parametric descriptive statistics, statistical models, inference and statistical tests. Non-parametric statistics is defined to be a function on a sample that has no dependency on a parameter whose interpretation does not depend on the population fitting any parameterised distributions[1]. Numerous methods exist for testing statistical hypotheses in various conditions. In some cases, the probability distribution of the population from which samples are drawn are known, for instance, if the population are assumed to be normal, then, the sample size is assumed to be sufficiently large to justify the assumption of normality, otherwise, the test of goodness of fit is carried out to ascertain the distribution of the data. In special cases, if sample sizes of a set of observations is small and the probability distribution of the populations from which samples are drawn are unknown; hence, only distribution free test statistic can be used; non-parametric methods. Thus, in most cases where the assumptions of parametric methods such as normality, homogeneity of variance, independency etc. are violated or not met, the non-parametric methods are usually preferred. This explains the appropriateness of Kruskal-Wallis test in place of One-Way Analysis of Variance in test of significance difference among treatments. These methods require that the populations from which the samples are drawn be continuous so that the probability of obtaining tied observations is at least theoretically zero [2]. Techniques or methods for performing two sample tests abound but the question is "which method(s) perform better and under what conditions do they perform better when dealing with independent samples?" To make an articulate attempt to answer these questions, there is need for proper and adequate comparative study of similar methods that can be used for the purpose of interest. Since the methods perform similar function and are widely used by researchers, there is a need for proper study of their strengths to determine the appropriate condition(s) under which each method performs optimally and which method is relatively more efficient and hence more powerful generally. In the determination of more effective statistical method, not just the null hypothesis should be of paramount interest but also the alternative hypothesis since the power of test plays an important role in the determination of effectiveness of statistical methods. The maximum value of power of

test is 1 and the least is zero which is non-negativity property. The higher the power of test is, the better the method, the lower the value and the less effective the method become.

**Material and Methodology**

**Mann-Whitney u-test** is used for determination of the likelihood that two samples/groups emanated from the same population/distribution[3].

The test statistic is:

$$Z = \frac{U - \mu_u}{\sigma_u} \qquad (1)$$

Then:

$$U = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1 \qquad (2)$$

where:

$n_1$ is the total number of the first group/observation.

$n_2$ is the total number of the first group/observation.

Then:

$$\mu_U = \frac{n_1 n_2}{2} \qquad (3)$$

is the mean and

$$\sigma_u = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}} \qquad (4)$$

is the standard deviation.

This Z-score is, as usual, compared at a given level of significance with an appropriate critical value obtained from a normal distribution table for a rejection or acceptance of the null hypothesis.

**Modified Intrinsically Ties Adjusted Mann-Whitney U Test** is used to check whether two samples could have been drawn from the same population/distribution [5].

The test statistic is:

$$\chi^2 = \frac{w^2}{var(w)} = \frac{\left(n_{2.}R_{\cdot 1} - n_{1.}R_{\cdot 2}\right)^2}{\left(n_{2.}R_1^{*2} + n_{1.}R_2^{*2} - 2R_{\cdot 1}R_{\cdot 2}\right)\left(\pi^+ + \pi^- - (\pi^+ - \pi^-)^2\right)} \qquad (5)$$

Where:

$n_1$ is the sample size of variable $x_1$

$n_2$ is the sample size of variable $x_2$

$R_1$ and $R_2$ are the respective sums of the ranks assigned to observations from populations $x_1$ and $x_2$ in the combined ranking of these observations from the two populations.

$\pi^+$,$\pi^-$are respectively the probabilities that observations or scores by subject from population $X_1$ is on the average greater than or less than observations or scores by subject from population $X_{2.}$

The test hypothesis will be:

$$H_0: \pi^+ - \pi^- = 0$$

vs

$$H_1: \pi^+ - \pi^- \neq 0$$

Reject $H_0$ at $\alpha$-level of significance if $\chi^2 \geq \chi^2_{1-\alpha;1}$; otherwise, accept.

**Two-sample kolmogorov–Smirnov test**

Kolmogorov-Smirnov two-sample test is a test of whether two independent samples have been drawn from the same population (or from populations with the same distribution)[5]. Their test statistic is:

$$D = maximum\left|S_{n_1}(X) - S_{n_2}(X)\right| \quad (6)$$

Where:

$S_{n_1}(X) = the\ observed\ cumulative\ step\ function\ of\ one\ of\ the\ samples.$

$S_{n_2}(X) = the\ observed\ cumulative\ step\ function\ of\ the\ other\ samples.$

$S_{n_1}(X) = {}^{K}/_{n_1}, where\ K = the\ number\ of scores\ equal\ to\ or\ less\ than\ X$

$S_{n_2}(X) = {}^{K}/_{n_2}, where\ K = the\ number\ of scores\ equal\ to\ or\ less\ than\ X$

$n_1\ is\ the\ total\ number\ of\ the\ first\ group.$

$n_2\ is\ the\ total\ number\ of\ the\ second\ group.$

The null hypothesis is rejected when the observed $D$ is equal or larger than the critical value.

Power of a statistical test is the probability of rejecting the null hypothesis when it is in fact false and should be rejected (i.e. the probability of not committing a type II error). Therefore, power of a test is (1-$\beta$) which is also known as the sensitivity[6]; where $\beta$ is the probability of committing type II error = error rate. Error rate is defined as the ratio of number of erroneous decision to number of replicate. That is:

$$\text{E.R} = \frac{Number\ of\ error\ output}{No\ of\ trials\ (Replicate)}$$

In this paper, Monte Carlo's Simulation techniques were used in the generation of data of different distributions and varying sample sizes ranging from 5 to 100 which were repeated 30 times for each sample size. In the simulation, sample sizes 5, 10, 20, 30, 50 and 100 were considered. In the paper, data from a known family of distributions; Gamma (4, 0.3) and Weibull (7, 3) were used.

**Algorithm for Monte Carlo Simulation**

Monte Carlo (MC) simulation is used to determine the performance of an estimator or test statistic under various scenarios [7]. The structure of a typical Monte Carlo exercise is as follows:

1. Specify the "Data Generation Process".
2. Choose a sample size N for the MC simulation.
3. Choose the number of times to repeat the MC simulation.
4. Generate a random sample of size N based on the Data Generation Process.
5. Using random sample generated in 4 above, calculate the statistic(s).
6. Go back to (4) and repeat (4) and (5) until desirable replicate is achieved.
7. Examine parameter estimates, test statistics, etc.

**Result**

**Algorithm for the Analysis**

The data analysis involves the following steps:

1. Pairs of simulate data of sizes n = 5, 10, 20, 30, 50, 100 from the two distributions; Gamma and Weibull.
2. For each sample size, n, replicated 30 times,

i. Calculate the values for test statistics for each of the methods.
ii. Reject/Accept the null hypothesis;
iii. Calculate the error rate = $\frac{times\ wrong\ decision\ is\ recorded}{number\ of\ replicate\ (30)}$
iv. Calculate the type I error or power of test = $1-p(type\ II\ error)$

The simulated data for Gamma and Weibull distribtuions are in the appendix A while for the test statistics and the p-value are in appendix B. From the simulated data using Monte Carlo simulation approach, the following results were obtained:

**Table 1: Error Rate of Mann-Whitney, MMWU and Two-sample K-S test Statistics**

| Family of Data | Test Statistic | 5 | 10 | 20 | 30 | 50 | 100 |
|---|---|---|---|---|---|---|---|
| Gamma (4,0.3) | Mann-Whitney | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0333 | 0.0333 |
| | MMWU | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | Two-Sample K-S | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0667 | 0.0667 |
| Weibull (7,3) | Mann-Whitney | 0.0333 | 0.1333 | 0.0000 | 0.0333 | 0.0000 | 0.0333 |
| | MMWU | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| | Two-Sample K-S | 0.0000 | 0.0000 | 0.0333 | 0.1000 | 0.0000 | 0.0333 |

Considering Mann-Whitney U test and MMWU; Mann-Whitney U test statistic is more suitable as the error rates of MMWU are significantly high for all the distributions considered.

Moreover, considering Mann-Whitney U test and Two-sample K-S; Mann-Whitney U test is better than Two-Sample K-S since the error rate of it is lower. For Weibull distribution, Two-sample K.S test is better than Mann-Whitney U-test for sample size of 5 and 10 and as the sample size increases Mann-Whitney U-test becomes more efficient than Two-Sample K-S test. Generally, the best statistical tool among the tests considered is Mann-Whitney U test followed by Two-Sample Kolmogorov-Smirnov test.

Table 2 can be computed from Table 1. Power of test is the sensitivity of a test statistic and the greater the value is, the more sensitive the test statistic becomes.

**Table 2: Power of Tests**

| Family of Data | Test Statistic | 5 | 10 | 20 | 30 | 50 | 100 |
|---|---|---|---|---|---|---|---|
| Gamma (4,0.3) | Mann-Whitney | 1 | 1 | 1 | 1 | 0.9667 | 0.9667 |
| | MMWU | 0 | 0 | 0 | 0 | 0 | 0 |
| | Two-Sample K-S | 1 | 1 | 1 | 1 | 0.9333 | 0.9333 |
| Weibull (7,3) | Mann-Whitney | 0.9667 | 0.8667 | 1 | 0.9667 | 0.9667 | 0.9667 |
| | MMWU | 0 | 0 | 0 | 0 | 0 | 0 |
| | Two-Sample K-S | 1 | 1 | 0.9667 | 0.9 | 1 | 0.9667 |

From Table 1, it can be observed that Mann-Whitney U-test and Two-sample K.S are more sensitive than MMWU test since it has higher power irrespective of the distribution of the data used.

For better understanding of sensitivity of the three test statistics, line chart of power of test is constructed as shown in figures 1 and 2 below.

Line chart can be used to show position of the strength or power of a test statistic, especially in statistical inference. This shows test statistic with higher power with the maximum power of 1.0.

**Graphical Illustration of power of test**

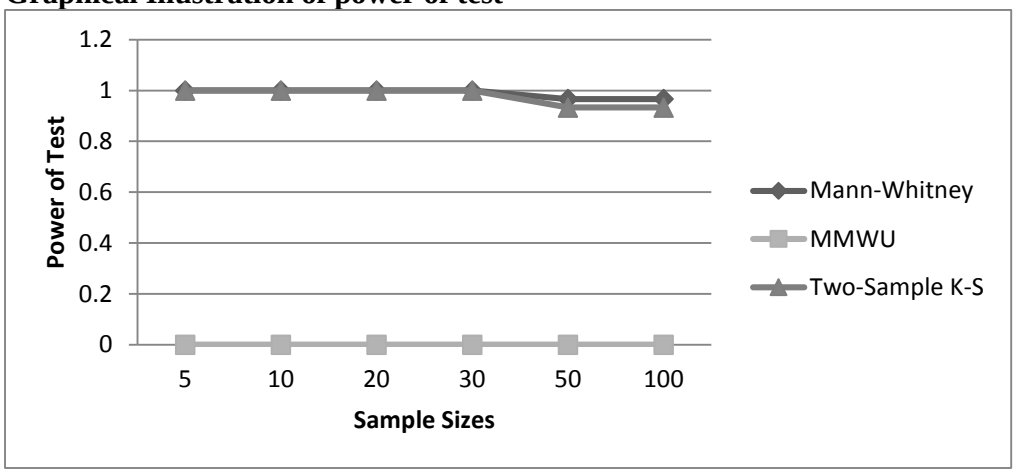


**Fig. 1: Power of Tests using Gamma Distribution**

As shown in Figure 1, it can be deduced that Mann-Whitney and Kolmogorov-Smirnov two-sample tests have the highest power irrespective of sample size which makes it better than MMWU. The modified method has considerably low power as sample size varies/increases.

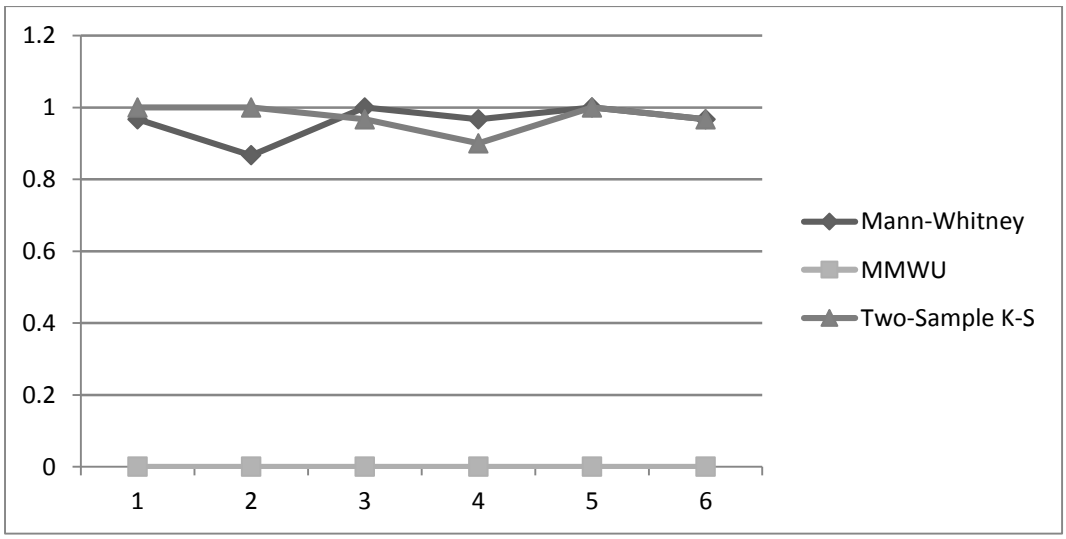


**Fig. 2: Power of Tests using Weibull Distribution**

Figure 2 revealed that Mann-Whitney and two-sample Kolmogorov-Smirnov have the same power at sample size 100 while from sample size 20 through 30 the Mann-Whitney U-test was

found to be more powerful. It was equally observed that the two-sample Kolmogorov-Smirnov test has better power than the Mann-Whitney U-test at sample points 5, 10 and 50.

**Summary and Conclusion**

We have in this paper presented a nonparametric statistical method for the analysis of two sample tests. Based on the result of the analysis used, it is observed that Mann-Whitney U test and Two-sample Kolmogorov-Smirnov test are more sensitive than Modified Mann-Whitney U-test (MMWU) since it has higher power.

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**APPENDIX** A

**Simulated Data of Weibull (7,3) for sample size of 5**

| 1 | Family A | Family B | 6 | Family A | Family B | 11 | Family A | Family B | 16 | Family A | Family B |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2.0549 | 2.14784 | | 2.96402 | 3.51324 | | 2.94196 | 2.07892 | | 3.90147 | 2.44272 |
| | 1.85224 | 2.75316 | | 2.91013 | 1.81929 | | 3.29378 | 3.2387 | | 2.10908 | 3.49174 |
| | 3.08328 | 2.80008 | | 3.45238 | 2.77946 | | 3.04343 | 2.58707 | | 1.94705 | 2.84907 |
| | 2.90837 | 2.35402 | | 3.27096 | 3.50724 | | 2.90073 | 2.99723 | | 3.2844 | 3.06377 |
| | 3.14121 | 2.47486 | | 2.89771 | 3.23779 | | 2.137 | 3.23329 | | 3.32377 | 3.30414 |
| 2 | 2.88806 | 3.13144 | 7 | 2.93772 | 3.0863 | 12 | 2.90932 | 2.87299 | 17 | 3.63141 | 3.52207 |
| | 2.77326 | 2.87969 | | 2.40444 | 2.46448 | | 3.13589 | 2.93109 | | 2.6874 | 2.33098 |
| | 3.19739 | 2.35186 | | 2.45472 | 2.52569 | | 2.17833 | 2.6693 | | 3.20825 | 1.22832 |
| | 2.07176 | 2.60132 | | 3.11643 | 2.74658 | | 2.21586 | 2.99436 | | 2.84293 | 2.87588 |
| | 1.93168 | 3.07957 | | 2.18975 | 2.70344 | | 2.82251 | 2.72761 | | 2.8962 | 3.54094 |
| 3 | 2.3961 | 2.76598 | 8 | 3.42744 | 3.04821 | 13 | 2.8056 | 3.24986 | 18 | 2.27389 | 2.90326 |
| | 3.19833 | 3.31849 | | 3.18529 | 3.55529 | | 2.6185 | 3.19856 | | 2.94375 | 3.20292 |
| | 2.30491 | 2.60751 | | 2.8518 | 3.36748 | | 3.08287 | 2.38237 | | 2.8712 | 2.70119 |
| | 2.57209 | 3.05334 | | 2.64509 | 2.85469 | | 3.19403 | 3.01101 | | 2.74751 | 3.12434 |
| | 2.53297 | 2.9933 | | 2.40766 | 3.00503 | | 2.38355 | 2.7196 | | 1.98065 | 2.3821 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 2.7355 | 2.2393 | 9 | 2.82601 | 3.02524 | 14 | 2.9674 | 3.09445 | | . | . |
| | 3.33363 | 3.41242 | | 2.6148 | 1.71342 | | 2.35219 | 3.26889 | | . | . |
| | 2.45461 | 2.87357 | | 3.43739 | 3.43751 | | 2.36213 | 3.96679 | | . | . |
| | 2.5648 | 3.06979 | | 3.30584 | 2.36777 | | 3.08225 | 2.71729 | | . | . |
| | 2.83251 | 2.34117 | | 1.99436 | 2.74342 | | 3.1595 | 2.87068 | | . | . |
| 5 | 2.57779 | 2.65905 | 10 | 2.53308 | 2.07577 | 15 | 3.18469 | 2.17382 | 30 | 2.78797 | 1.79462 |
| | 3.33877 | 2.48479 | | 1.97652 | 2.99061 | | 3.43874 | 3.21076 | | 1.21927 | 1.96139 |
| | 3.11277 | 3.11663 | | 3.03554 | 2.96309 | | 3.10281 | 1.85871 | | 2.54097 | 3.56915 |
| | 3.68913 | 2.94917 | | 2.63108 | 2.2902 | | 3.4404 | 2.65302 | | 2.0308 | 3.43897 |
| | 3.38483 | 3.15906 | | 2.68229 | 3.05492 | | 2.90121 | 3.00366 | | 2.88939 | 2.36357 |

**Simulated Data of G(4,0.3) for sample size of 5**

| 1 | Family A | Family B | 6 | Family A | Family B | 11 | Family A | Family B | 16 | Family A | Family B |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.540693 | 1.132631 | | 0.358768 | 2.823391 | | 0.936434 | 0.536449 | | 0.850696 | 1.043107 |
| | 1.099999 | 0.252437 | | 0.784115 | 0.5173 | | 1.426788 | 0.606461 | | 1.762179 | 2.177001 |
| | 1.257383 | 0.530388 | | 2.281035 | 1.112356 | | 0.772563 | 2.050076 | | 0.286225 | 3.211577 |
| | 1.350839 | 0.666464 | | 1.511695 | 0.187971 | | 1.855367 | 0.84629 | | 0.440526 | 1.015095 |
| | 0.822252 | 1.973966 | | 1.730911 | 1.321935 | | 1.224994 | 0.936048 | | 1.282424 | 1.298899 |
| 2 | 0.388482 | 1.164518 | 7 | 0.930303 | 0.387684 | 12 | 1.607881 | 0.809028 | 17 | 0.54221 | 0.451363 |
| | 3.367172 | 2.664101 | | 2.490846 | 0.779647 | | 1.655428 | 0.979611 | | 2.76167 | 1.723086 |
| | 1.615149 | 1.2452 | | 2.677828 | 0.920208 | | 0.890015 | 0.887636 | | 1.067012 | 2.417414 |
| | 1.652795 | 2.030195 | | 1.745883 | 2.201069 | | 0.986908 | 1.319078 | | 0.477154 | 1.742978 |
| | 0.987986 | 0.780936 | | 1.840938 | 0.61363 | | 1.124742 | 1.157486 | | 0.744717 | 1.153614 |
| 3 | 2.032877 | 1.637004 | 8 | 0.431434 | 2.036976 | 13 | 1.513215 | 0.774746 | 18 | 1.307757 | 2.038412 |
| | 0.743287 | 1.946726 | | 0.414789 | 1.874285 | | 0.395753 | 0.468649 | | 0.79132 | 1.727142 |
| | 1.741365 | 1.070626 | | 2.36039 | 0.751185 | | 0.819668 | 0.781594 | | 0.797857 | 1.449444 |
| | 0.617396 | 0.666522 | | 0.559442 | 1.497564 | | 1.561971 | 1.220831 | | 1.096657 | 1.06419 |
| | 0.756069 | 1.882293 | | 1.706953 | 1.129871 | | 0.909478 | 0.224561 | | 1.411843 | 0.643461 |
| 4 | 0.58093 | 0.35945 | 9 | 1.333053 | 1.22878 | 14 | 0.669428 | 0.87758 | | . | . |
| | 1.286098 | 0.97924 | | 0.65221 | 2.670646 | | 0.59603 | 1.572339 | | . | . |
| | 0.943381 | 2.477932 | | 1.62474 | 1.541398 | | 1.402757 | 1.547851 | | . | . |
| | 1.287931 | 1.295778 | | 2.186741 | 0.828445 | | 0.932162 | 0.955364 | | . | . |
| | 1.821823 | 1.25112 | | 0.742813 | 2.59682 | | 1.19045 | 2.021954 | | . | . |
| 5 | 1.447551 | 0.588709 | 10 | 0.75928 | 0.630048 | 15 | 0.350237 | 1.420993 | 30 | 1.346355 | 1.04301 |
| | 0.772286 | 1.272155 | | 1.065302 | 2.526689 | | 0.31228 | 1.365376 | | 1.211608 | 1.109491 |
| | 1.424602 | 1.242772 | | 1.035784 | 0.737097 | | 1.617787 | 1.18782 | | 0.624364 | 1.601369 |
| | 0.966316 | 0.347502 | | 0.655443 | 0.679389 | | 0.64783 | 0.432679 | | 0.954253 | 0.819451 |
| | 1.202835 | 1.173304 | | 0.635915 | 0.570286 | | 1.690238 | 0.430992 | | 1.289154 | 0.319329 |

**Simulated Data of G(4,0.3) for sample size of 10**

| 1 | Family A | Family B | 4 | Family A | Family B | 7 | Family A | Family B | 10 | Family A | Family B |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1.47499 | 2.16983 | | 1.88868 | 2.08685 | | 3.22103 | 1.83574 | | 2.32448 | 2.0299 |
| | 1.42161 | 2.39375 | | 2.18657 | 3.36969 | | 1.95863 | 2.58577 | | 3.64263 | 3.64746 |
| | 2.48126 | 1.25254 | | 1.90092 | 2.17343 | | 2.44478 | 1.56001 | | 1.81066 | 1.61465 |
| | 3.21448 | 3.0012 | | 2.62925 | 2.64181 | | 1.85193 | 1.73784 | | 1.68763 | 1.54577 |
| | 2.18517 | 1.18478 | | 3.73508 | 2.57196 | | 2.63755 | 2.43675 | | 2.84849 | 2.0025 |
| | 2.92477 | 1.72976 | | 2.05133 | 3.0852 | | 1.33498 | 2.15427 | | 2.1945 | 2.61388 |
| | 1.45767 | 2.26609 | | 2.13 | 1.8455 | | 2.17335 | 1.59713 | | 1.46567 | 1.92751 |
| | 2.59934 | 1.11223 | | 1.84952 | 2.77504 | | 1.17087 | 1.4318 | | 2.31214 | 2.4049 |
| | 1.88754 | 2.22914 | | 2.4633 | 2.67339 | | 1.42922 | 3.73844 | | 1.72302 | 3.44591 |
| | 1.46279 | 2.76588 | | 1.67666 | 1.44516 | | 2.63589 | 2.12359 | | 1.31912 | 2.79673 |
| 2 | 1.94833 | 2.122 | 5 | 2.34784 | 1.67795 | 8 | 2.92256 | 1.48792 | | . | . |
| | 1.72293 | 1.19955 | | 1.28589 | 1.78727 | | 2.31797 | 2.19734 | | . | . |
| | 2.12779 | 2.02204 | | 2.03819 | 2.16127 | | 1.91942 | 1.37619 | | . | . |
| | 1.9467 | 1.46287 | | 3.01982 | 1.50966 | | 2.91263 | 1.91007 | | . | . |
| | 2.47781 | 2.48328 | | 2.75024 | 1.47947 | | 1.79161 | 2.2411 | | . | . |
| | 3.24088 | 1.66386 | | 2.10093 | 2.27131 | | 1.51698 | 1.38885 | | . | . |
| | 1.65636 | 4.05627 | | 1.46282 | 1.81715 | | 2.48521 | 2.39244 | | . | . |
| | 2.01588 | 2.38656 | | 2.94184 | 1.53951 | | 3.22881 | 1.52286 | | . | . |
| | 1.82542 | 3.30058 | | 1.70085 | 1.73702 | | 1.54559 | 2.7478 | | . | . |
| | 1.798 | 2.40882 | | 2.11124 | 1.64646 | | 2.29959 | 1.6585 | | . | . |
| 3 | 1.33483 | 3.20387 | 6 | 2.35779 | 1.82536 | 9 | 2.07225 | 1.63325 | 30 | 1.66275 | 2.71397 |
| | 2.78723 | 2.1343 | | 2.24233 | 2.24667 | | 2.25963 | 1.77296 | | 1.1359 | 1.84029 |
| | 2.02814 | 2.32298 | | 2.11648 | 1.65336 | | 1.56957 | 1.8824 | | 3.34375 | 2.46886 |
| | 2.11915 | 1.55359 | | 1.6537 | 1.69109 | | 2.95141 | 1.49451 | | 2.74775 | 2.49164 |
| | 1.82279 | 2.56755 | | 1.83126 | 1.62992 | | 1.8214 | 4.30432 | | 2.12868 | 3.28533 |
| | 2.04075 | 2.20246 | | 2.03558 | 2.18532 | | 2.83388 | 2.29046 | | 3.12887 | 2.29621 |
| | 2.30211 | 4.10863 | | 1.76504 | 2.28915 | | 2.18556 | 1.85333 | | 1.33987 | 2.81134 |
| | 1.7381 | 1.87024 | | 2.29264 | 2.29546 | | 1.65249 | 1.53898 | | 3.01046 | 2.22307 |
| | 1.91443 | 1.36457 | | 2.49943 | 2.92259 | | 2.41737 | 2.1812 | | 2.8458 | 3.17786 |
| | 4.27942 | 2.10988 | | 2.13913 | 2.38827 | | 1.84901 | 1.60819 | | 1.51609 | 1.97865 |

## Appendix B

**Table A: Test Statistic and P-values of Weibull Distribution using Mann-Whitney U-Test for Sample size 5**

| S/N | Chi-Square | P-Value | Remark | Accuracy of Decision |
|---|---|---|---|---|
| 1 | 0.522 | 0.6015 | Accept Ho | Correct Decision |
| 2 | 0.522 | 0.6015 | Accept Ho | Correct Decision |
| 3 | 1.776 | 0.0758 | Accept Ho | Correct Decision |
| 4 | 0.104 | 0.9168 | Accept Ho | Correct Decision |
| 5 | 1.358 | 0.1745 | Accept Ho | Correct Decision |
| 6 | 0.104 | 0.9168 | Accept Ho | Correct Decision |
| 7 | 0.9168 | 0.4647 | Accept Ho | Correct Decision |
| 8 | 1.149 | 0.2506 | Accept Ho | Correct Decision |
| 9 | 0.313 | 0.754 | Accept Ho | Correct Decision |

| | | | | |
|---|---|---|---|---|
| 10 | 0.522 | 0.6015 | Accept Ho | Correct Decision |
| 11 | 0.104 | 0.9168 | Accept Ho | Correct Decision |
| 12 | 0.522 | 0.6015 | Accept Ho | Correct Decision |
| 13 | 0.522 | 0.6015 | Accept Ho | Correct Decision |
| 14 | 1.149 | 0.2506 | Accept Ho | Correct Decision |
| 15 | 1.776 | 0.0758 | Accept Ho | Correct Decision |
| 16 | 0.104 | 0.9168 | Accept Ho | Correct Decision |
| 17 | 0.522 | 0.6015 | Accept Ho | Correct Decision |
| 18 | 1.149 | 0.2506 | Accept Ho | Correct Decision |
| 19 | 0.104 | 0.9168 | Accept Ho | Correct Decision |
| 20 | 0.94 | 0.3472 | Accept Ho | Correct Decision |
| 21 | 0.313 | 0.754 | Accept Ho | Correct Decision |
| 22 | 0.522 | 0.6015 | Accept Ho | Correct Decision |
| 23 | 1.776 | 0.0758 | Accept Ho | Correct Decision |
| 24 | 1.776 | 0.0758 | Accept Ho | Correct Decision |
| 25 | 0.94 | 0.3472 | Accept Ho | Correct Decision |
| 26 | 0.522 | 0.6015 | Accept Ho | Correct Decision |
| 27 | 1.984 | 0.0472 | Reject Ho | Correct Decision |
| 28 | 1.358 | 0.1745 | Accept Ho | Correct Decision |
| 29 | 0.731 | 0.4647 | Accept Ho | Correct Decision |
| 30 | 0.313 | 0.754 | Accept Ho | Correct Decision |

**Table b: Test Statistic and P-values of Weibull Distribution using MMWU Test for size 5**

| S/N | Chi-Square | P-Value | Remark | Accuracy of Decision |
|---|---|---|---|---|
| 1 | 5.20833333 | 0.022479 | Reject $H_o$ | Incorrect Decision |
| 2 | 9.300595238 | 0.002291 | Reject $H_o$ | Incorrect Decision |
| 3 | 5.00801282 | 0.02523 | Reject $H_o$ | Incorrect Decision |
| 4 | 6.853070175 | 0.008849 | Reject $H_o$ | Incorrect Decision |
| 5 | 5.00801282 | 0.02523 | Reject $H_o$ | Incorrect Decision |
| 6 | 5.42534722 | 0.019846 | Reject $H_o$ | Incorrect Decision |
| 7 | 6.20039683 | 0.012772 | Reject $H_o$ | Incorrect Decision |
| 8 | 5.07305195 | 0.024301 | Reject $H_o$ | Incorrect Decision |
| 9 | 5.20833333 | 0.022479 | Reject $H_o$ | Incorrect Decision |
| 10 | 5.00801282 | 0.02523 | Reject $H_o$ | Incorrect Decision |
| 11 | 5.20833333 | 0.022479 | Reject $H_o$ | Incorrect Decision |
| 12 | 5.20833333 | 0.022479 | Reject $H_o$ | Incorrect Decision |
| 13 | 6.20039683 | 0.012772 | Reject $H_o$ | Incorrect Decision |
| 14 | 9.300595238 | 0.002291 | Reject $H_o$ | Incorrect Decision |
| 15 | 5.00801282 | 0.02523 | Reject $H_o$ | Incorrect Decision |
| 16 | 6.20039683 | 0.012772 | Reject $H_o$ | Incorrect Decision |
| 17 | 5.00801282 | 0.02523 | Reject $H_o$ | Incorrect Decision |
| 18 | 5.7444853 | 0.016541 | Reject $H_o$ | Incorrect Decision |
| 19 | 5.07305195 | 0.024301 | Reject $H_o$ | Incorrect Decision |
| 20 | 5.20833333 | 0.022479 | Reject $H_o$ | Incorrect Decision |
| 21 | 9.300595238 | 0.002291 | Reject $H_o$ | Incorrect Decision |
| 22 | 5.7444853 | 0.016541 | Reject $H_o$ | Incorrect Decision |
| 23 | 6.853070175 | 0.008849 | Reject $H_o$ | Incorrect Decision |
| 24 | 1.18E+01 | 5.81E-04 | Reject $H_o$ | Incorrect Decision |
| 25 | 5.42534722 | 0.019846 | Reject $H_o$ | Incorrect Decision |
| 26 | 5.07305195 | 0.024301 | Reject $H_o$ | Incorrect Decision |
| 27 | 9.300595238 | 0.002291 | Reject $H_o$ | Incorrect Decision |
| 28 | 5.20833333 | 0.022479 | Reject $H_o$ | Incorrect Decision |
| 29 | 5.20833333 | 0.022479 | Reject $H_o$ | Incorrect Decision |
| 30 | 5.20833333 | 0.022479 | Reject $H_o$ | Incorrect Decision |

**Table c: Test Statistic and P-values of Weibull Distribution using Two K.S Test for size 5**

| S/N | Chi-Square | P-Value | Remark | Accuracy of Decision |
|---|---|---|---|---|
| 1 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 2 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 3 | 0.8 | 0.052 | Accept $H_o$ | Correct Decision |
| 4 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 5 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 6 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 7 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 8 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 9 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 10 | 0.2 | 1 | Accept $H_o$ | Correct Decision |
| 11 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 12 | 0.2 | 1 | Accept $H_o$ | Correct Decision |
| 13 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 14 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 15 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 16 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 17 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 18 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 19 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 20 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 21 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 22 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 23 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 24 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 25 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 26 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 27 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 28 | 0.6 | 0.251 | Accept $H_o$ | Correct Decision |
| 29 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |
| 30 | 0.4 | 0.752 | Accept $H_o$ | Correct Decision |

**Table d: Test Statistic and P-values of Gamma Distribution using K.S for Sample 10**

| S/N | Chi-square | P-value | Remark | Decision |
|---|---|---|---|---|
| 1 | 0.3 | 0.238 | Accept Ho | Correct Decision |
| 2 | 0.3 | 0.238 | Accept Ho | Correct Decision |
| 3 | 0.25 | 0.452 | Accept Ho | Correct Decision |
| 4 | 0.15 | 0.959 | Accept Ho | Correct Decision |
| 5 | 0.25 | 0.452 | Accept Ho | Correct Decision |
| 6 | 0.15 | 0.959 | Accept Ho | Correct Decision |
| 7 | 0.2 | 0.739 | Accept Ho | Correct Decision |
| 8 | 0.3 | 0.238 | Accept Ho | Correct Decision |
| 9 | 0.3 | 0.238 | Accept Ho | Correct Decision |
| 10 | 0.25 | 0.452 | Accept Ho | Correct Decision |
| 11 | 0.2 | 0.739 | Accept Ho | Correct Decision |
| 12 | 0.2 | 0.739 | Accept Ho | Correct Decision |
| 13 | 0.25 | 0.452 | Accept Ho | Correct Decision |
| 14 | 0.2 | 0.739 | Accept Ho | Correct Decision |
| 15 | 0.25 | 0.452 | Accept Ho | Correct Decision |
| 16 | 0.15 | 0.959 | Accept Ho | Correct Decision |

| | | | | |
|---|---|---|---|---|
| 17 | 0.1 | 1 | Accept Ho | Correct Decision |
| 18 | 0.2 | 0.739 | Accept Ho | Correct Decision |
| 19 | 0.15 | 0.959 | Accept Ho | Correct Decision |
| 20 | 0.25 | 0.452 | Accept Ho | Correct Decision |
| 21 | 0.25 | 0.452 | Accept Ho | Correct Decision |
| 22 | 0.3 | 0.238 | Accept Ho | Correct Decision |
| 23 | 0.15 | 0.959 | Accept Ho | Correct Decision |
| 24 | 0.3 | 0.238 | Accept Ho | Correct Decision |
| 25 | 0.2 | 0.739 | Accept Ho | Correct Decision |
| 26 | 0.35 | 0.112 | Accept Ho | Correct Decision |
| 27 | 0.25 | 0.452 | Accept Ho | Correct Decision |
| 28 | 0.2 | 0.739 | Accept Ho | Correct Decision |
| 29 | 0.2 | 0.739 | Accept Ho | Correct Decision |
| 30 | 0.3 | 0.238 | Accept Ho | Correct Decision |

**Table e: Test Statistic and P-values of Weibull Distribution using Mann-Whitney U-Test**

| S/N | Chi-square | P-value | Remark | Decision |
|---|---|---|---|---|
| 1 | 2.495 | 0.0126 | Reject Ho | Incorrect Decision |
| 2 | 0.756 | 0.4497 | Accept Ho | Correct Decision |
| 3 | 2.343 | 0.0191 | Reject Ho | Incorrect Decision |
| 4 | 1.436 | 0.1509 | Accept Ho | Correct Decision |
| 5 | 0.983 | 0.3258 | Accept Ho | Correct Decision |
| 6 | 2.041 | 0.0413 | Reject Ho | Incorrect Decision |
| 7 | 0.227 | 0.8206 | Accept Ho | Correct Decision |
| 8 | 1.058 | 0.2899 | Accept Ho | Correct Decision |
| 9 | 0.68 | 0.4963 | Accept Ho | Correct Decision |
| 10 | 0.529 | 0.5967 | Accept Ho | Correct Decision |
| 11 | 0.151 | 0.8798 | Accept Ho | Correct Decision |
| 12 | 0.076 | 0.9397 | Accept Ho | Correct Decision |
| 13 | 1.058 | 0.2899 | Accept Ho | Correct Decision |
| 14 | 0.907 | 0.3643 | Accept Ho | Correct Decision |
| 15 | 0.076 | 0.9397 | Accept Ho | Correct Decision |
| 16 | 0.907 | 0.3643 | Accept Ho | Correct Decision |
| 17 | 1.285 | 0.1988 | Accept Ho | Correct Decision |
| 18 | 0.907 | 0.3643 | Accept Ho | Correct Decision |
| 19 | 0.151 | 0.8798 | Accept Ho | Correct Decision |
| 20 | 0.832 | 0.4057 | Accept Ho | Correct Decision |
| 21 | 0.302 | 0.7624 | Accept Ho | Correct Decision |
| 22 | 0.454 | 0.6501 | Accept Ho | Correct Decision |
| 23 | 0.68 | 0.4963 | Accept Ho | Correct Decision |
| 24 | 2.343 | 0.0191 | Reject Ho | Incorrect Decision |
| 25 | 1.361 | 0.1736 | Accept Ho | Correct Decision |
| 26 | 0.076 | 0.3643 | Accept Ho | Correct Decision |

| | | | | |
|---|---|---|---|---|
| 27 | 0.907 | 0.3643 | Accept Ho | Correct Decision |
| 28 | 0.076 | 0.9397 | Accept Ho | Correct Decision |
| 29 | 0.227 | 0.8206 | Accept Ho | Correct Decision |
| 30 | 1.058 | 0.2899 | Accept Ho | Correct Decision |

**Table f: Test Statistic and P-values of Weibull Distribution using MMWU**

| S/N | Chi-square | P-value | Remark | Decision |
|---|---|---|---|---|
| 1 | 1.77E+01 | 2.56E-05 | Reject Ho | Incorrect Decision |
| 2 | 10.41666667 | 0.001248831 | Reject Ho | Incorrect Decision |
| 3 | 1.62E+01 | 5.57E-05 | Reject Ho | Incorrect Decision |
| 4 | 1.17E+01 | 6.29E-04 | Reject Ho | Incorrect Decision |
| 5 | 10.72501073 | 0.001056971 | Reject Ho | Incorrect Decision |
| 6 | 1.41E+01 | 1.72E-04 | Reject Ho | Incorrect Decision |
| 7 | 10.03613007 | 0.001534993 | Reject Ho | Incorrect Decision |
| 8 | 1.09E+01 | 9.88E-04 | Reject Ho | Incorrect Decision |
| 9 | 10.33484911 | 0.001305418 | Reject Ho | Incorrect Decision |
| 10 | 10.1999184 | 0.001404469 | Reject Ho | Incorrect Decision |
| 11 | 10.01602564 | 0.00155184 | Reject Ho | Incorrect Decision |
| 12 | 10.0040016 | 0.001562004 | Reject Ho | Incorrect Decision |
| 13 | 1.09E+01 | 9.88E-04 | Reject Ho | Incorrect Decision |
| 14 | 10.61120543 | 0.001124044 | Reject Ho | Incorrect Decision |
| 15 | 10.0040016 | 0.001562004 | Reject Ho | Incorrect Decision |
| 16 | 10.61120543 | 0.001124044 | Reject Ho | Incorrect Decision |
| 17 | 1.13E+01 | 7.72E-04 | Reject Ho | Incorrect Decision |
| 18 | 10.61120543 | 0.001124044 | Reject Ho | Incorrect Decision |
| 19 | 10.01602564 | 0.00155184 | Reject Ho | Incorrect Decision |
| 20 | 10.50861707 | 0.001188191 | Reject Ho | Incorrect Decision |
| 21 | 10.06441224 | 0.001511608 | Reject Ho | Incorrect Decision |
| 22 | 10.1461039 | 0.001446065 | Reject Ho | Incorrect Decision |
| 23 | 10.33484911 | 0.001305418 | Reject Ho | Incorrect Decision |
| 24 | 1.62E+01 | 5.57E-05 | Reject Ho | Incorrect Decision |
| 25 | 11.48897059 | 0.000700104 | Reject Ho | Incorrect Decision |
| 26 | 10.0040016 | 0.001562004 | Reject Ho | Incorrect Decision |
| 27 | 10.61120543 | 0.001124044 | Reject Ho | Incorrect Decision |
| 28 | 10.61120543 | 0.001124044 | Reject Ho | Incorrect Decision |
| 29 | 10.03613007 | 0.001534993 | Reject Ho | Incorrect Decision |
| 30 | 1.09E+01 | 9.88E-04 | Reject Ho | Incorrect Decision |